

Probabilistic Borehole Stability Analysis using the First Order Reliability Method

Jinsong Huang^{a*} and Sau-Wai Wong^b

^a *Discipline of Civil, Surveying & Environmental Engineering
Priority Research Centre for Geotechnical Science & Engineering
The University of Newcastle, Callaghan, NSW 2308, Australia*

^b *Centre for Offshore Research and Engineering (CORE)
Department of Civil and Environmental Engineering, National University of Singapore
jinsong.huang@newcastle.edu.au

Abstract

In a typical borehole stability analysis, near-wellbore rock stress changes and deformation are modelled to provide a recommendation for optimum drilling mud weight. Such deterministic approach to borehole stability analysis is well established. However, deterministic methods usually account for subsurface and operational uncertainties indirectly by adopting conservative input parameters. The results may lead to suboptimal drilling operation design. The First Order Reliability Method (FORM) is used to consider the uncertainties in the design parameters, such as in-situ stresses, mud pressure and rock properties. The probabilistic model provides a view of the operational risk in terms of the probability of failure as a function of uncertainties. Hence, it allows for a more informed decision making process and provides a better risk management strategy.

Keywords: Probabilistic analysis, Wellbore stability, First Order Reliability Method, Risk management

1. Introduction

Wellbore instability problems are often encountered in the drilling and construction of deep wells in the oil and gas industry. In the well construction process, the borehole may collapse or substantial volume of drilling fluid may be lost due to fracturing of rock formation, which can lead to a number of severe drilling operation problems such as lost circulation of drilling fluid, stuck drill string and consequent fishing, sidetracking and reaming operations, even a complete loss of wellbore. Generally, the occurrences of the wellbore instability related problems significantly add to the already high cost of well construction. It is estimated that at least 10% of the average well budget is used on unplanned operations resulting from wellbore instability. This cost may approach one billion dollars per year worldwide.

A number of deterministic techniques have been developed to predict optimal operational parameters such as mud weights or drilling trajectories (e.g., Huang et al. 2012, Huang et al. 2011), in which either for the sake of simplicity or for lack of information, it is assumed that the geomechanical and operational parameters are the same throughout a material domain. However, in many cases, due to the intrinsic inhomogeneous nature of the rocks, the minimal exposure of the rock mass around a borehole, and the need to extrapolate available information over a depth range, the geomechanical parameters such as the in-situ stresses, pore pressure, and rock strength are inevitably poorly assessed, as the required data necessary to compute their values are often not available. Furthermore, models that describe the relationships between field measured data and the require parameters for modelling are poorly calibrated. In some cases, technological or operational constraints make it impossible to acquire the information necessary to overcome these problems. An additional problem relates to the intrinsic uncertainty or error associated with each measurement. Thus, the uncertainty involved in the wellbore stability problem is largely due to the lack of knowledge about the properties of the rock mass and the in situ stresses rather than due to the inherent randomness in those parameters.

Because of the uncertainty involved in the wellbore stability analysis, the use of some ‘averaged values’ for the input parameters in the deterministic approaches may not fully capture the operation risk and challenges due to wellbore instability. A better understanding of the impact of these uncertainties would provide a more informed decision-making process and help pin-point where to invest for gathering of additional data to reduced uncertainties. It is feasible to employ probabilistic methods to quantify the impacts of uncertainties on wellbore stability predictions. Although probabilistic methods have frequently been used in the oil industry, e.g. to estimate the expected hydrocarbon recovery and the economic value of a development project, their application to wellbore stability is relatively new and not widely adopted in practice

Based on reliability indices, de Fontoura et al. (2002) presented three analytical methods for evaluating the influence of input parameter uncertainties on wellbore failure modes. Their results were compared and shown to agree well with that of Monte Carlo method. Sheng et al. (2006) proposed a numerical geomechanical modeling of wellbore failures, in which a statistical method was incorporated. Mud pressure was singled out as the most influential input variable affecting wellbore deformation. Using Fast Lagrangian Analysis of Continua simulator, and Latin Hyperbole Sampling technique, a range of safe mud weights that guaranteed stable wellbore were predicted. Moos et al. (2003) adopted and modified a Quantitative Risk Assessment (QRA) approach, and analyzed both the collapse pressure and the lost circulation pressure to derive a mud window, both at a single depth and over an entire open hole. Muller et al. (2009) presented the influence of both spatial variability of mechanical and hydraulic properties, and the simple variability of initial conditions on two-dimensional borehole stability analysis. In that study, comparisons of results obtained from both stochastic and deterministic analyses, show that the stability window (range of internal pressures corresponding to stability conditions) obtained with a deterministic analysis correspond to a higher failure probability when compared with the stability window obtained with a stochastic analysis. Similarly, critical mud pressures have been estimated through probabilistic wellbore stability analysis where Monte Carlo sampling techniques were used to capture uncertainties in in-situ stresses, wellbore trajectory, cohesion, friction angle, Poisson ratio (Al-Ajmi and Al-Harthi 2010; Al-Khayari et al. 2016; Sheng et al. 2006; Udegbuma et al. 2014), pore water pressure and rock strength (Sheng et al. 2006). Wellbore trajectory was determined as the most influential parameter to avoid wellbore collapse (Al-Khayari et al. 2016), while other critical parameters impacting on wellbore stability were identified as friction angle, cohesion and maximum horizontal stress (Al-Ajmi and Al-Harthi 2010). Eshiet and Sheng (2018) show that stochastic analyses can be used to improve results derived from deterministic models. The incorporation of stochastic techniques in the evaluation of wellbore instability indicates that margins of the safe mud weight window are adjustable and can be extended considerably beyond the limits of deterministic predictions.

It is noted however, almost all previously mentioned studies used Monte Carlo simulations. The only except is de Fontoura et al. (2002) who compared the results obtained from the First Order Second Moment (FOSM) method, the First Order Reliability Method (FORM) and the Monte Carlo simulations. The Monte Carlo simulation method suffers from its inefficiency of modeling small probability of failure and the FOSM suffers from giving inconsistent results when performance function is formulated in different ways. FORM is the preferable method when a quick assessment of the implications of uncertainties is desirable, which is usually the case for prototype designs. In this paper, FORM is adopted and tested in the analysis of borehole instability. The approach is formulated in such a way that can easily exploit readily available solvers (for example, the built-in solver in Excel spreadsheet). The random variables which can be considered are design mud pressure, in situ stresses, pore pressure and the shear strength of rocks. A wide range of probability distributions are implemented in the program including Normal, Lognormal, Tanh, Beta, Extreme Value, Exponential, Uniform, Triangle, Weibull, Gamma and Pert. The implemented program has been validated against Monte Carlo simulations.

2. The First Order Reliability Method

2.1 Review of FORM

The first order reliability method (FORM) is a process which can be used to estimate the probability of failure of systems involving multiple random variables with given probability density functions, in relation to a “limit state” function, which is a function that separates the failure or unsatisfactory domain from the safe domain. The conventional FORM based on the Hasofer-Lind reliability index (Hasofer and Lind 1974), β_{HL} , assumes that the mean values of random variables lie on the safe side of the limit state function. The method then obtains the reliability index, which is related to the minimum distance, in directional standard deviation units, between the mean values and the limit state surface. Commonly used software packages (e.g. Excel and Matlab) are easily adapted to perform the optimization. Once

the reliability index (the distance between the means and the closest failure point) has been determined, the method assumes a “first order” limit state function tangent to the β_{HL} contour, and the probability of failure, p_f follows from

$$p_f = 1 - \Phi(\beta_{HL}) \quad (1)$$

If dealing with two random variables, the “first order” assumption results in a straight line limit state function, in which case p_f is the volume under the bi-variate probability density function on the failure side of the line. A similar approach is used for multiple random variable problems.

Each reliability analysis requires a limit state function, which defines safe or unsafe performance. Limit states could relate to strength failure, serviceability failure, or anything else that describes unsatisfactory performance. The limit state function, g , is customarily defined

$$\begin{aligned} g(X_1, X_2, \dots, X_N) &\geq 0 \longrightarrow \text{Safe} \\ g(X_1, X_2, \dots, X_N) &< 0 \longrightarrow \text{Failure} \end{aligned} \quad (2)$$

where X_1, X_2, \dots, X_N are the input random variables. An advantage of the FORM is that the result it gives is not affected by the form of the limit state function. For example, the limit state function could be defined as the resistance minus the load, the factor of safety minus one, the logarithm of the factor of safety or some other algebraic combination.

The limit state function can sometimes be determined directly from theory, or for more complex systems, the response surface method (e.g., Melchers, 1999) needs to be used. The basic idea of this method is to approximate the limit state boundary by an explicit function of the random variables, and to improve the approximation via iterations

In detail, the determination of β_{HL} is an iterative process defined by

$$\beta_{HL} = \min_{g=0} \sqrt{\left\{ \frac{X'_i - \mu_i^N}{\sigma_i^N} \right\}^T [R]^{-1} \left\{ \frac{X'_i - \mu_i^N}{\sigma_i^N} \right\}} \quad i=1,2,\dots,n \quad (3)$$

where $\{(X'_i - \mu_i^N) / \sigma_i^N\}$ is the vector of n random variables reduced to standard normal space and R is the matrix of correlations between the standard normal variables.

For the purpose of illustrating how FORM applies to borehole stability analysis, the example below employs a built-in solver in Excel.

3. Examples

A vertical wellbore is used as example to show how the FORM works. The following limit state function (e.g., Huang et al. 2012) is adopted.

$$g = p_m - \frac{1}{2}(3\sigma_H - \sigma_h)(1 - \sin \phi') + c' \cos \phi' - u \sin \phi' \quad (4)$$

where p_m is mud pressure, σ_H is major horizontal in situ stress, σ_h is minor horizontal in situ stress, ϕ' is internal friction angle of rock, c' is cohesion of rock, and u is pore pressure.

The statistical data for the wellbore stability analyses are list in Table 1, where all random variables are assumed to be Normally distributed. The probability of failure based on the limit state function Eq. (4) is calculated and list in Fig. 1. Also shown in Fig. 1 are the results obtained from direct Monte Carlo simulations using one thousand million simulations. It can be seen from Fig. 1 that FORM provides consistent results with Monte Carlo simulations. It is noted that FORM requires much less computational time than Monte Carlo simulations.

Further analyses are conducted to investigate the influence of probability distribution on the probability of failure. It is assumed that all random variables in Table 2 is Lognormally distributed. The results are compared to the ones of Normal distribution in Fig. 2. It can be seen from Fig. 2 that assuming Lognormal distribution provide more conservative estimations compared to Normal distribution. It is also noted that Lognormal distribution cannot take negative values which is true for the mechanical properties of rocks.

Table 1 Statistical data for wellbore stability analyses

Parameter	Unit	Distribution	Mean	Standard Deviation
p_m	MPa	deterministic	22.0	0
σ_H	MPa	Normal	40.0	4.0
σ_h	MPa	Normal	35.2	3.52
u	MPa	Normal	20.0	2.0
$\tan(\phi')$	/	Normal	0.4663	0.04663
c'	MPa	Normal	15.9	1.59

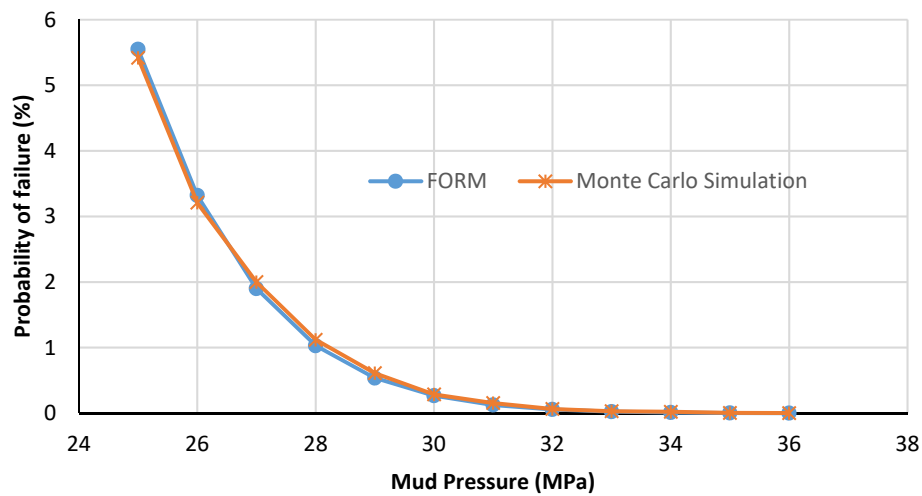


Fig. 1 Results of wellbore stability analyses

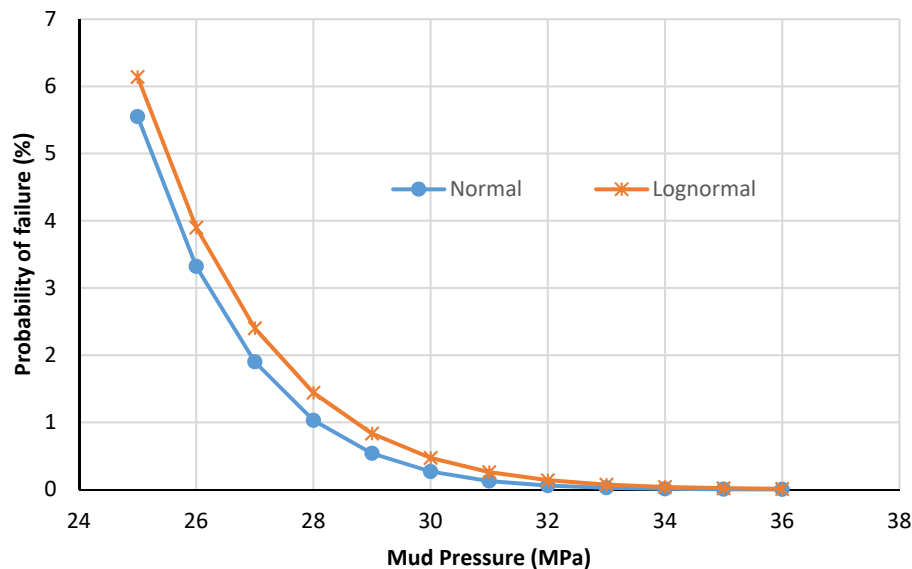


Fig. 2. Comparison of the results of Normal and Lognormal distributions

The second example involves a real case where the uncertainties of input parameters have been assessed based on field measurements. The initial assessments of the parameters that should be modelled as random variables are shown in Table 2. Both triangular and uniform distributions are proposed to model the uncertainties in the parameters. The statistical parameters are listed in Table 3. FORM is then used to estimate the probability of failure under various mud pressures. The results are shown in Fig. 3. It can

be seen from Fig. 3 that uniform distribution significantly overestimates probability of failure than triangular distribution.

Table 2 Uncertainty estimation of input parameters

Parameter	Unit	Most likely value	Uncertainty in estimation ($\pm\%$)	Range of magnitude
σ_H	sg	1.8	10	1.62-1.98
σ_h	sg	1.5	5	1.43-1.58
u	sg	1.05	30	0.74-1.37
ϕ'	degree	30°	20	24° -36°
c'	sg	0.5	50	0.25-0.75

Table 3 Statistical data for wellbore stability analyses

Parameter	Distribution	
	Triangular	Uniform
σ_H	1.62, 1.8, 1.98	1.62, 1.98
σ_h	1.43, 1.5, 1.58	1.43, 1.58
u	0.74, 1.05, 1.37	0.74, 1.37
ϕ' (in radian)	0.420, 0.524, 0.628	0.420, 0.628
c'	0.25, 0.5, 0.75	0.25, 0.75

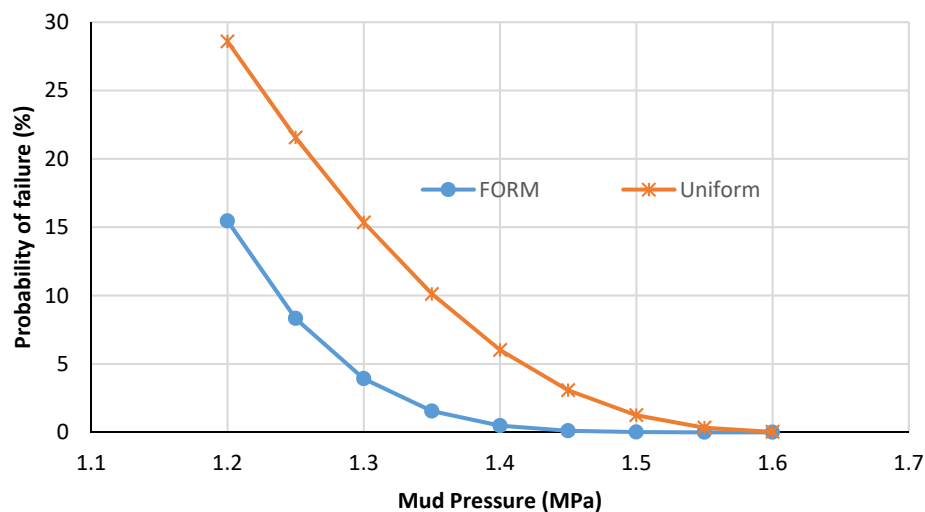


Fig. 3. Comparison of the results of triangular and uniform distributions

4. Conclusions

Deterministic techniques do not readily account for risks or uncertainties. In deterministic analysis, the wellbore is either declared 'stable' or 'unstable'. Stochastic techniques incorporate variations and uncertainties due to influencing factors including variabilities in design and operating parameters. This paper presents the use of First Order Reliability Method for borehole stability analysis. It is relatively straight forward to implement as demonstrated in the example which employed a built-in solver in Excel to assess the stability of a vertical wellbore. A more comprehensive application of FORM in combination with a non-linear (elasto-plastic) borehole stability model has been coded for field application and it is a subject of future paper. The goal is to promote a more widespread application of probabilistic methods in petroleum geomechanics and allow for a more robust evaluation of related uncertainties and operation risks. Such a perspective would lead to better decision; for example, the range of mud pressure (safe drilling mud weight window) might be widen without incurring unacceptable risk in borehole instability.

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